

TRAFFIC SIMULATION MODELLING FOR HIGHWAY SYSTEMS

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for the Degree of

MASTER OF TECHNOLOGY

by

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to the

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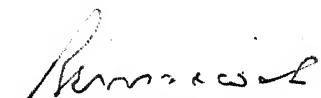
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CERTIFICATE

This is to certify that the thesis entitled, "TRAFFIC SIMULATION MODELLING FOR HIGHWAY SYSTEMS", by Mr.A.Raghuram Reddy, in partial fulfilment of the requirements for the degree of Master of Technology of the Indian Institute of Technology, Kanpur, is a record of bonafide work carried out by him under my supervision and guidance. The work embodied in this thesis has not been submitted elsewhere for a degree.

August , 1988.


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ABSTRACT

The Traffic Simulation Model outlined in this work, simulates the flow of mixed traffic on two lane and multilane highway systems, including intersections.

The Indian highway network is such that there are number of intersections at grade, where different highways meet or cross each other. The traffic flow passing through them assumes complex behaviour pattern, affecting considerably the flow process on the road system. Hence, to capture the flow behaviour on the entire highway system it is necessary to simulate the traffic flow through intersections.

The objective of this study was to develop a submodel for traffic flow through intersections and incorporate the same into existing Indian Traffic Simulation Model (ITSM) without disturbing its initial structure. Simulation runs were made to test the flow logic. A few simulation experimental runs were also made to study the model output for certain levels of input parameters. Necessary modifications to ITSM with respect to intersections include:

- (a) introduction of intersection code.
- (b) to reduce the speeds of the vehicles approaching

intersection, this is achieved in the model by reducing their local speeds as they are about to enter an intersection.

(c) to wait in the queue at intersections if sufficient gap is not available between cross flow vehicles.

(d) to clear the intersection if enough gap is available.

The model outputs the performance of simulated vehicles both on intersections and road sections. The events that are recorded with regard to an intersection consist of identification no. of vehicle, vehicle type, time of entry, waiting time, time to clear, exit time etc.

Since this is the first attempt in this direction, detailed sensitivity analysis was not taken up. The model system when fully developed after validation and calibration, will have wide applications on the Indian Highway Network System.

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1. CHAPTER

INTRODUCTION

1.1 Background:

The simulation model, presented in this study, simulates the flow of mixed traffic on two lanes and multilane highway systems, including intersections. The model outputs the performance of simulated vehicles both on intersections and road sections.

The national and state highways running across the length and breadth of our country are mostly two lane wide, with 7 metre wide pavements with or without shoulders on each side. As, considerable proportion of traffic moves on this road network, study, of traffic flow on them assumes pivotal importance. Due to highly heterogenous nature of vehicles on Indian roads, the study of traffic characteristics and behaviour on road sections adds more complexities.

1.2 Traffic Flow:

A traffic flow model should be able to evaluate

- Speed flow relations
- Capacity of road at various levels of service; and

- effectiveness of alternate remedial aids in terms of traffic flow and traffic safety.

This can be achieved by simulating the behaviour of vehicles on road system.

The traffic flow composition on Indian roads is mostly heterogeneous in character with both fast and slow moving vehicles having wide variations in speeds and dimensions. Fast moving vehicles such as cars, trucks and buses interact with a number of slow moving vehicles like animal driven vehicles etc., Two wheeled motor vehicles like scooters/motorcycles also considerably affect the flow of other vehicles. The variation in dimensions of different categories of vehicles affects the traffic flow behaviour (overtaking and passing manoeuvres). Wide vehicles such as trucks, cars and bullock carts etc., occupy almost full lane while moving at their free speeds. Whereas, two wheelers like scooters and motorcycles occupy very little width and hence two or more of them can travel in one lane. Trucks, buses and cars require more clear width of road than animal driven vehicles.

It is very much difficult to study the flow pattern of the mixed traffic unless the interactions between the vehicles are studied in detail by a realistic simulation

model. Most of the national highways in this country are only two lane wide carrying a large volume of traffic with faster vehicles coming across a number of desired overtakings or passings.

1.3 Traffic Simulation Modelling:

Simulation is resorted to when the system in study is not amenable to analytic techniques and direct experimentation with the system is impractical. Simulation is a technique for representing a dynamic system by a model and experimenting with the model in order to gain information about the system.

The traffic flow on road system represents a complex logical behaviour pattern, which cannot be studied by analytical or empirical modelling. Simulation modelling can help to describe the behaviour of the traffic stream as vehicles traverse a specified section of the road.

The simulation model should thus have overtaking and passing logic's. The model should be composed of a series of submodels, both deterministic and stochastic in nature to describe the free flowing traffic and interaction between vehicles. The relevancy of the model is dependent upon the accuracy of the submodels the manner in

which the submodels are linked and the inclusion of relevant submodels.

There have been various attempts in the past (A simulation model of two lane and four lane highways for Indian traffic conditions. Marwah B.R, 1983) to simulate the traffic on two lane roads. This has ventured in the evolution of various models; the most sophisticated and comprehensive traffic simulation model is the VTI model, developed by the Swedish road and traffic institute.

1.3.1 About The VTI Model:

The VTI model is the outcome of the work done by Gynnerstedt et al., (1977,1979) at VTI. This is the first discrete event traffic simulation model with a long history of development and rigorous validation. The model considers homogenous motorized traffic. The VTI model is based on the large scale data collection in connection with overtaking behaviour and desired speed distribution.

The desired speed is related to road width, speed limit, horizontal curvature and gradient. There are thirty two different types of overtaking maneuvers considered. The roadway is divided into homogenous sections called "blocks". Four categories of vehicles are considered in this model. Though these four classes of vehicles vary

with regard to speed, lengths and widths they all use full traffic lane for movement, which has made the formulation easy that there will be one vehicle at any point on a lane. The scanning is event oriented and the most tedious part of the model is the large amount of field work required for the input data.

As it is the case, the model has its limitations. Such as, it cannot be applied directly to Indian traffic which constitutes heterogeneous traffic composition, with wide variations in speed, power-weight ratio, acceleration/deceleration rates etc. The model does not consider the effect of intersections on the traffic flow.

Besides its usability the limitations has necessitated to modify the model without disturbing the basic structure of the Swedish model and extending it to be implemented for Indian road networks. The research work carried out at I.I.T. Kanpur, India, has resulted in the development of an Indian Traffic Simulation Model (ITSM) for implementation on Indian roads.

1.3.2 Indian Traffic Simulation Model (ITSM):

The ITSM has the capability to simulate 10 different vehicle types which include fast and even slowest vehicles such as bullock carts etc. These ten vehicle types

are grouped into four vehicle groups and used in the model. The categorised vehicle types into particular vehicle groups are represented in the model as per table 1.

Vehicle group	Group name	Vehicle type
1	Car	Car, tempo, Wagon etc.,
2	HMV	Bus, truck, Tractor, Heavy trucks etc.,
3	Two Wheelers	Scooters, Motor cycles etc.,
4	ADV	Horse carts, Bullock carts etc.,

Table 1.0

The dynamics of traffic interaction and the effect of road geometry on traffic operations have been captured through a discrete event based flow logic and programmed in SIMULA using JSP (Jackson Structured Programming) technique.

There are many submodels such as :-

- Basic desired speed
- Power/Weight ratio

- Gap acceptance probabilities
- Free speed

These submodels have been calibrated with extensive field data. The model has been validated for different levels of traffic flow on plain, rolling and hilly terrains for two lane and four lane roads. The model results has shown close agreement with the field observations for all the stretches considered. The simulation model thus possesses the potential for analyzing complex traffic behaviour on Indian roads.

The model provides adequate information about those factors relevant in assessing the effects on traffic operations, when road attributes are changed etc., It also provides insight for the planners to identify problem locations and for taking up necessary measures.

1.4 Problem Definition:

The Indian Traffic Simulation Model is able to simulate the flow of heterogeneous traffic on road sections with a sufficient level of confidence. However, the Indian highway systems has most of the intersections on grade. To assess the traffic flow performance the flow behaviour of vehicles through intersections also need to be studied, in detail. The vehicles may have to wait at intersection to get

way. The delay caused to a vehicle at the intersection depends upon

- Flow level in the stream
- Queue length and position of vehicle in the queue
- Cross flow levels and their headway distributions.

The flow system on the intersection is thus quite complex as the flow of vehicles on crossing roads also need to be studied. The simulation of vehicles on the road system having intersections will help in assessing the performance measures of the overall system and also the remedial measures for the intersection system.

It is proposed in this study to simulate the flow of vehicles on the road system with intersections.

1.5 Scope of the work:

The scope of the present work consists of developing a submodel for intersection simulation and to incorporate the same in the ITS model. The computer runs are to be made to test the logic. The model along with the newly developed submodel should be able to simulate the flow behaviour of traffic stream on road sections as well as intersections.

The future work relates to calibrate and validate the model against field experimentation.

2. CHAPTER

Traffic flow Modelling

2.1 Introduction:

A traffic simulation model describes the behaviour of the traffic system by considering in detail the behaviours of individual vehicles as they traverse a specified section of road. As mentioned, there are number of models both analytical and simulation which are used to study the behaviour pattern of traffic, but the model specified in this work (ITSM) is most advanced and comprehensive. This traffic simulation model is programmed in language called SIMULA 67 (O.J.Dahl, B.Myhrhaug, K.Nygaard, 1971) which is an extension of programming language ALGOL60. In the development of the model Jackson Structured Programming (JSP) technique is used for system and programming work. JSP is a problem solution method, programming language independent, software and hardware independent improves programme reliability and reduces programme maintenance costs dramatically (Ingevaldsson, 1980).

On the basis of the road geometry, speed limit and the presence of other vehicles on the road which prevents the movement of individual vehicles to travel at their speeds splits the traffic simulation model into two parts.

1.Free flow model : In which the vehicle speeds are

governed by geometry only.

2. Interaction model : In which the vehicle speeds are governed by interactions between the individual vehicles of the traffic stream.

2.2 Programming Language; SIMULA 67:

The traffic simulation program is written in SIMULA 67 language, which is an extension of ALGOL60. SIMULA language is designed to provide simulation facilities without losing the advantage of a powerful general purpose language.

SIMULA (G M Birtwistle et.al., 1973) consists of additional features like record oriented dynamic memory allocation, reference or pointer structures, sets and queues, text and character handling, sequential and direct access input-output, quasi parallel sequencing and processing (event) oriented simulation capabilities. It is designed in a way which makes it easy to produce perspicuous, well structured readable and secure programs. SIMULA67 is well adapted to structure programming methodology.

SIMULA is more than just a notation for describing computer processes - it also provides a framework for concept creation and is a tool of thought in problem analysis. SIMULA has been designed to trap as many errors as

possible during compile time. DEC 10/20 SIMULA version has also a very powerful on line debugging system called SIMDDT.

In SIMULA an object is a single entity which combines action and data associated with the component of a system. An object in general consists of three parts:-

- A heading identifying an object
- A data structure
- An action pattern.

A CLASS declaration provides all these properties which are common to every object of the class and which are not subjected to change during the object's life span. A CLASS declaration serves as a fixed enblock description.

SIMULA language is provided with a predefined system CLASS SIMULATION, which contains all the necessary concepts for queues and active and passive components. When the identifier SIMULATION is prefixed to a block, the objects of this block can make use of the concepts of SIMULATION. Such as, joining the queues, activating or passivating the necessary process, referencing the objects in the queue etc., within CLASS SIMULATION, inner to this there are three classes which are of prime importance.

PROCESS : Which is used to prefix classes whose objects

have active or passive phases.

LINK : Which is used to prefix classes whose objects are to be placed in and removed from queues.

HEAD : Which serves as the head of the queue.

PROCESS is itself prefixed by LINK, so that the PROCESS objects have queue membership attributes.

2.3 Process for simulation program:

The traffic simulation model which was developed by VTI consists of two processes:-

a. Vehicle generator process - It is a vehicle generator process, as soon as the vehicles are generated the PROCESS CLASS concept allots them individual driver vehicle attributes. These objects are defined by parameters like identification number, vehicle type, basic desired speed, power-weight ratio etc. Traffic attributes include coordinates at entry and exit, direction of travel, arrival time etc. This process also activates the vehicles at their starting times. This vehicle generator process is represented in the program by PROCESS CLASS GENERATOR PROCESS.

b. Equipage Process - PROCESS CLASS VEH

This process describes the behaviour pattern of each individual vehicle generated in the above process as they

traverse along the road section. It describes all the possibilities that a vehicle can have as it moves on the road like moving freely, following a vehicle, overtaking a vehicle or being overtaken, lane changes etc. This process is the kernel of the simulation model. As a vehicle comes across so many possibilities as it moves on the road section especially in mixed traffic conditions, it places considerable demand on program structures. As the vehicle moves from one block to other the times at which the next event of each vehicle will occur are calculated. The events are then executed in chronological order. The ordinary cycle for an arbitrary vehicle is:-

- (i) Predict the time of next event - PREDICTNEXTETIME
- (ii) Await the predicted time - HOLD
- (iii) Move the vehicle in time and space - DRIVE

2.4 Road description:

The road is represented in the simulation program as a series of road blocks and a sight distance function in each direction of travel. Each road block object is homogeneous with regard to road geometry (i.e. road width, surface type, auxillary lane, longitudinal slope, horizontal curvature) and traffic regulations (speed limit, overtaking

restriction). The road blocks are generated from the main simulation program and is represented in the program by LINK CLASS ROADBLOCK. Each roadblock generated will be homogenous with respect to above attributes and are placed in a series by the system procedure LINK.

2.5 Free flow and Interacting models:

Each vehicle in the traffic stream has its own basic desired speed at which it likes to move on the road, this basic desired speed at which vehicle tries to move is restrained by the road geometry or speed limit and presence of other vehicles on the road.

On the basis of this the traffic simulation model is divided into two parts. The first part in which the vehicle speed is limited by road geometry only is called "Free flow model". The second one in which the vehicle speed is governed by the presence of other vehicles on the road is called "Interaction model".

2.5.1 Free flow model:

The concept of basic desired speed is essential to a traffic simulation model. This is the speed at which a vehicle wishes to travel. But it is prevented in doing so by the following factors:

- Road width
- Horizontal curvature
- Speed limit
- Gradient

For each vehicle the speed is determined by considering the above three factors and the effect of gradient is superimposed to obtain the actual speed.

The basic objective in the development of this model involves calibration of a distribution for basic desired speed, for individual class of vehicles. Let this be denoted by V_0 and V_{0i} a sample from V_0 and V_{0m} denote the median value of V_0 .

Firstly, the effect of road width is considered on this distribution, the resulting distribution be denoted by V_1 and V_{1m} and V_{1i} carry their original meanings with respect to new distribution. Similarly the effect of horizontal curvature and speed limit results in the distributions,

$$V_2 - V_{2m}, V_{2i}$$

$$V_3 - V_{3m}, V_{3i} \text{ respectively.}$$

Analytically, it can be represented with function f describing the effect of factor 'i' on the median value V_{i-m} of V_{i-1} .

$$V_{1m} = f_1(V_{0m}, W)$$

$$V_{2m} = f_2(V_{1m}, R)$$

$$V_{3m} = f_3(V_{2m}, S)$$

Where, 'W' is the road width, 'R' the radius of curvature and 'S' the speed limit. The various steps in which the basic desired speed of a vehicle is subjected to constraints to adjust its speed is represented as shown in fig.2.1.

In order to reduce the calculation of the speed relationships, various transformations were used such that the speed difference between V_0 and V_3 is the same for all the speeds. i.e. the curves for the basic desired speeds of vehicles are transformed into a family of parallel curves. The calibration constant 'Q' is such that V_3 is approximately distributed as V_{0m}^Q , minus a shift 'D' which is dependent on all the three factors.

$$V_{0m}^Q - V_{3m}^Q = V_{0i}^Q - V_{3i}^Q = D$$

The transformations on the speed distribution functions can be represented diagrammatically as in figures 2.2(a) and 2.2(b).

The road description in the model has given as a series of homogenous block, with constant roadwidth, speed limit, horizontal curvature, gradient and overtaking restrictions throughout the length of the block. A change in

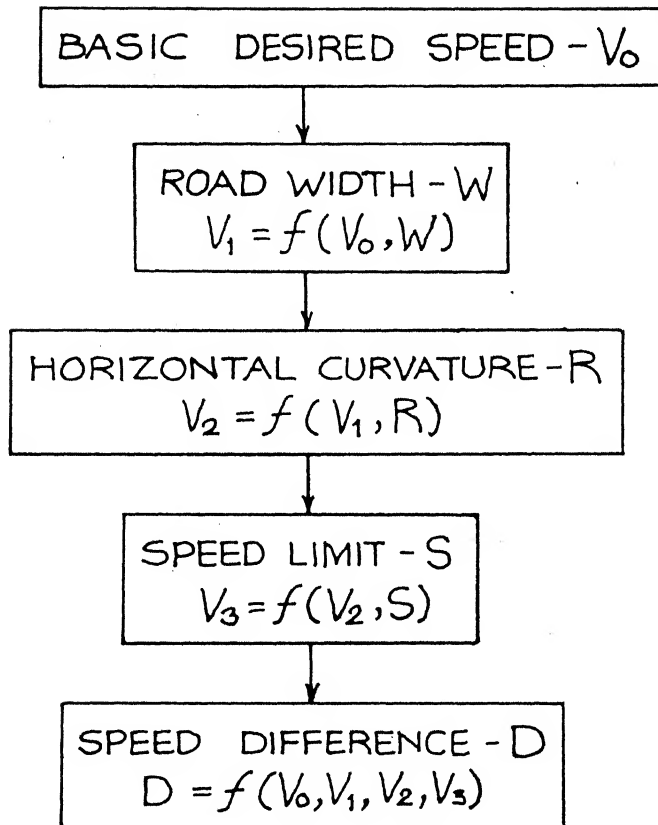
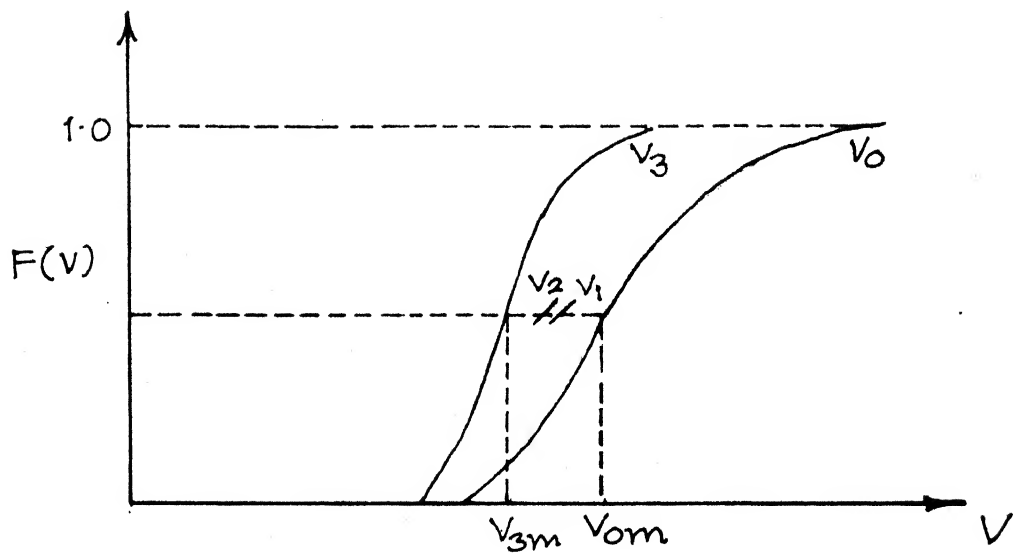


FIG. 2.1 - SPEED GEOMETRY RELATIONSHIP



(a)

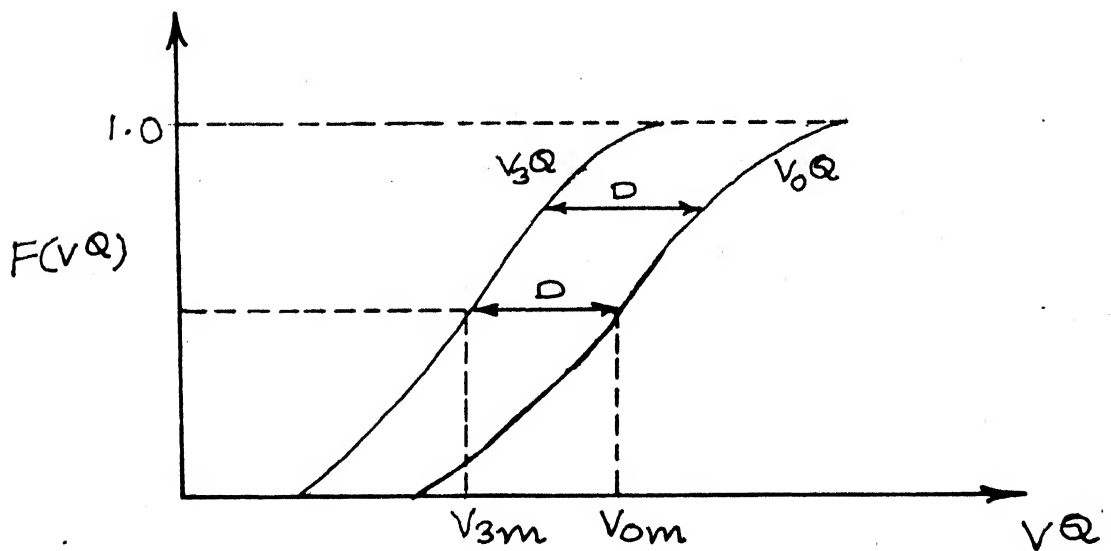


FIG. 2.2 - TRANSFORMATIONS ON THE DISTRIBUTION FUNCTIONS

any one of these parameters will mark the beginning of a new block or the block limit. The fig.2.3 shows the speed V_3 which is dependent on road geometry, changing its profile abruptly at the end of each block.

To come over this problem of abrupt change in speed profile at the end of each block limit necessitated the concept of "Influence block". Creation of this influence block ensures a gradual decrease or increase in speed or acceleration of the vehicle at the point of discontinuity as shown in fig.2.4.

Each type of vehicle is assigned a specific distribution of power to weight ratio. These ratio's are derived from observations on gradient using the following expression:

$$P = \frac{V_0^2 - V_1^2}{2S} + gh/S + C_r(V_0 + V_1)/2 + f(C_a, V_0, V_1)$$

where

- V_0 - Speed at the start of the section
- V_1 - Speed at the end of the section
- S - Length of the section
- h - Vertical height climbed
- C_r - Constant of rolling friction
- C_a - Constant of air friction
- g - Acceleration due to gravity

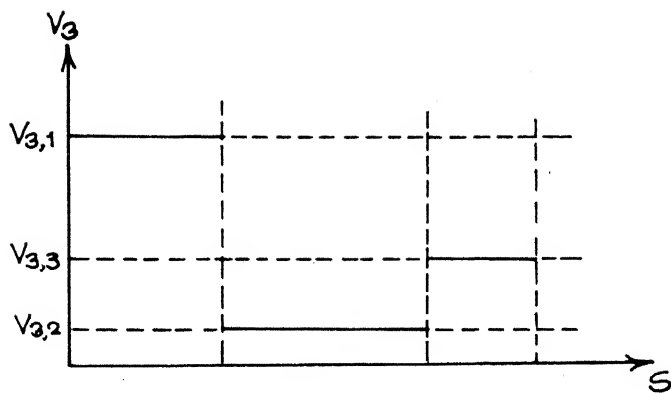


FIG. 2-3

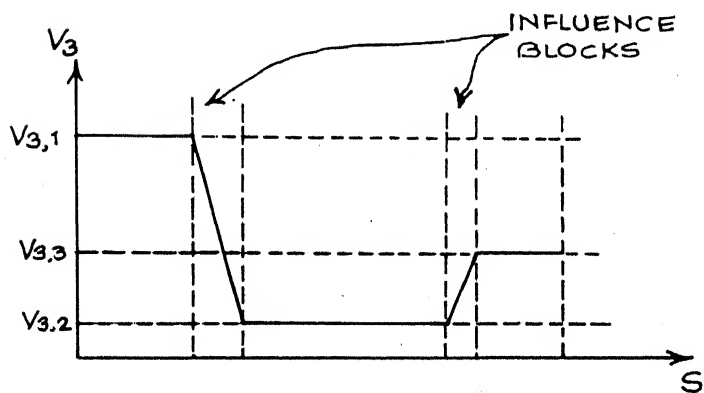


FIG. 2-4

CONTINUITY OF SPEED PROFILE INFLUENCE BLOCKS

The 'p' values are used to govern the ' V_3 ' distribution in order to incorporate the speed reduction on hills and also to govern the rate of acceleration both on the hills and on the top.

The effect of gradient on the speed distribution after considering the total effects of road width, horizontal curvature and speed limit is superimposed to obtain the actual speed at which the vehicle is traversing a road section.

The free block speed is the velocity of the vehicle in a specified block, as the traffic is divided into 10 types and each type has a specified speed distribution, power-weight ratio, rolling and air resistance co-efficients, the free speed of the vehicle (V_{3i}) for a block is determined from the following equation.

$$V_{3i}^Q = V_{0i}^Q - (V_{0m}^Q - V_{3m}^Q)$$

where

V_{3i} = Desired speed in the homogenous block

V_{0i} = Assigned basic desired speed of the
vehicle

V_{0m} = Basic desired speed of the median
vehicle

V_{3m} = Block speed for the median vehicle

2.5.2 Interaction model:

Vehicles interact with each other as they move over the road sections. A vehicle is said to be interacted by surrounding vehicles when it is constrained by another vehicle or a group of vehicles or it is overtaking/passing another vehicle or a group of vehicles. Interaction of vehicles in traffic with each other restrains them to move at the desired speed of each vehicle type. Interaction model is the most important of the traffic simulation process in that, it affects the entire behaviour of the traffic stream.

2.5.2(a) Track/lane classification:

The VTI model divides the road width into three parts or tracks/lanes for each direction of travel.

- (i) Track 2 : The usual lane in which traffic moves in one direction
- (ii) Track : The lane which is used by overtaking vehicles
- (iii) Track : Hard side shoulders or extra lanes which is used by vehicles in passing operation.

In India, most of the highway roads have unpaved shoulders which are very rarely used for passing manoeuvres. The following diagram in fig.2.5 represents the road width with three tracks for each direction of travel.

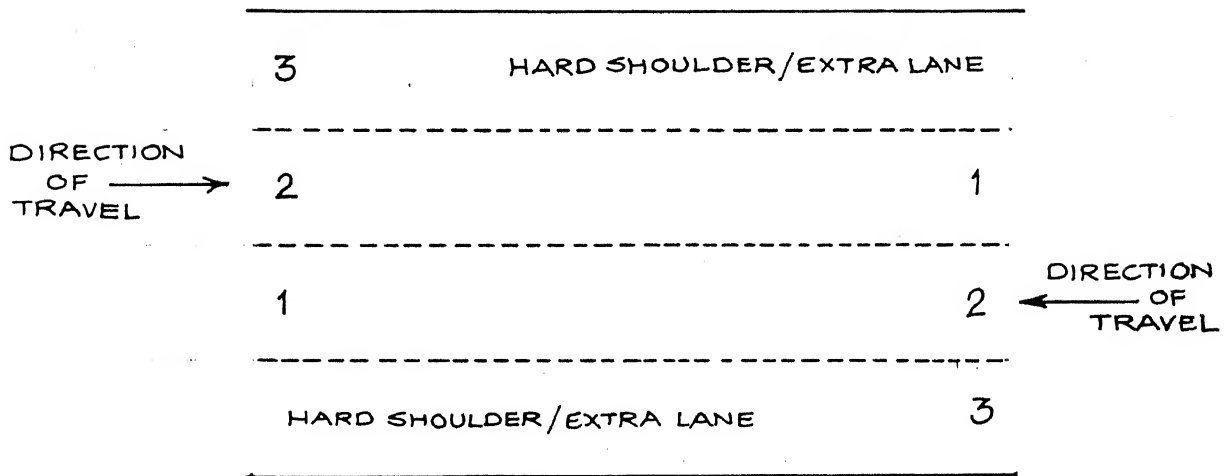


FIG. 2.5 - CLASSIFICATION OF TRACS/LANES IN THE MODEL.

2.5.2(b) Events or Decision points:

The VTI model is an event based one, the primary event takes place while crossing road blocks. Similar additional events will take place where sight distances change and also where interaction takes place between vehicles. Events occur during interaction wherever the status of the vehicle changes. The status of the vehicle is updated as soon as it passes the following points:-

- A decision point for flying overtaking
- A decision point for accelerative overtaking
- A point when the overtaking vehicle is in level with the overtaken vehicle
- A point where an overtaking finishes
- A point where an oncoming vehicle crosses a vehicle desiring to overtake
- A block limit

Since the present lays emphasis on the simulation of traffic at intersections where in the possibility of any overtaking (flying/accelerated) is ruled out, for detailed description about flying or accelerated overtakings report no. 321 and 43 of the National Road and Traffic Research Institute (Sweden) or the detailed report on Indian Traffic Simulation can be consulted.

2.5.2(c) Reference to Interacting vehicles:

The two lane highways of our country are 7.0m. wide with unpaved shoulders on both sides. The composition of the traffic is such that the vehicles generally do not remain in the specified lane. So, in case of mixed traffic flow, at times, two vehicles may be traveling at their free speeds in one lane without any interaction like 2 scooters or a scooter and an animal drawn vehicle(ADV). Thus, for determining the operation mode of a vehicle one is not interested in the vehicles which are ahead or behind it but in that vehicles which are likely to interact.

In VTI model whenever reference is to be made to a vehicle ahead or behind the current vehicle the following procedure is adopted.

(i) calculate reference (R) to vehicle (ahead or behind) the current vehicle (C) in lane X.

(ii) Test is made whether the type of referred vehicle(R) interacts with the type of current vehicle. The following table 2 gives those combinations of vehicle types that do not interact in the same lane. In case of no interaction vehicle ahead or behind or 'R' is tested for interaction with current vehicle 'C'.

Type of current vehicle	Reference vehicle (R)
Cars	Scooters
HMV	Scooters
Scooters	Scooters, HMV, Cars, ADV
ADV	Scooters

Table-2.0

2.5.2(d) Prediction of events:

The model is event based. The event list consists of details about the events currently predicted to take place next. Events are assumed to occur at calculated times. At each event, the data are updated and a particular event is generated from among the possible consecutive event types. Events are executed in a chronological order.

The ordinary cycle for an arbitrary vehicle consists of :-

- (i) The time of occurrence of next event is predicted -
PREDICTNEXTTIME
- (ii) Await for the predicted time. It may happen that during this phase, other surrounding vehicles may interact with the current vehicle and the prediction

of next event time in the above may be not correct. A prediction of the new earlier event time is then made.

(iii) The vehicle is moved in time and space i.e. the vehicle attributes are updated.

2.6 Validity of a traffic simulation model:

The description outlined above represents the manner in which a real life road system is modeled in a computer.

A simulation model becomes useful only if it enables us to learn something about the system. This is achieved firstly, by calibrating the model against field data and secondly, by validating the model to check for its potential to meet different field situations. It may also be necessary to calibrate the values of some parameters in the light of simulation results during validation exercises. The properties to be used for validation should be such that they represent the output of the observed and simulated means and standard deviations helps in quantifying how good or bad the comparisons were.

The results obtained from the simulation model (ITSM) has proved to be in close comparison with the field data. The efficiency of the model limits to the road sections

without any intersections in between. The necessary modifications incorporated into the model through present study, considers the presence of intersections and their effect on traffic behaviour passing through intersection.

3.0 CHAPTER

TRAFFIC FLOW MODEL FOR INTERSECTION

3.1 Introduction:

The Indian Traffic Simulation Model (ITSM) for two lane and four lane highways as existing simulates the flow of different vehicle types on the road sections of different vehicle types on the road sections of different geometrics. The model has been calibrated and validated for plain, rolling and hilly terrains.

The Indian highway system is such that there are a number of intersections at grade, where different highways meet or cross each other. To capture the flow behaviour on the entire highway system, it is necessary to simulate also the behaviour through intersections. The inclusion of intersections in the road system is attempted in this study. The ITSM is being modified in such a manner that its existing data structure is retained and the effect of simulation of vehicles through intersections is also incorporated. The model system, as developed in this study, is a first attempt in this direction. This model system when fully developed will have wide applications on the Indian Highway Network System.

3.2 About the model:

This model is integrated with the free flow and vehicle interaction models of ITSM. The model simulates the complete behaviour of the vehicles in terms of speed changes, turning movements, delays, waiting etc. as they pass through an intersection. The output of this model is helpful in determining the flow characteristics like speed, delay etc. through the intersections.

A vehicle when approaches an unsignalized intersection, the driver desires to take a left turn, right turn or move straight depending upon his destination and the layout of intersection. The intersection may be a T-junction or a crossing with four or more approaches. The flow behaviour of the vehicle entering an intersection is affected by the characteristics of intersections, and also by the traffic streams moving in different directions. In certain situations the vehicle may clear an intersection without any delay whereas in someother cases it may have to decelerate or stop and look for a gap between the vehicles in cross flow.

The flow logic for the intersection is relatively complex compared to the road sections. The simulation of the vehicles is attempted in this model considering the intersection characteristics and the traffic

flow characteristics of different sections.

3.3 Intersection environment:

The geometric characteristics of the intersection that are of interest in the flow process include the length of the intersection along the road under simulation, the length of the intersection at merging/crossing flows, the number of lanes and of geometrics. The road width on the section close to intersection is generally more than that of the road stretch. A driver approaching an intersection may psychologically reduce the speed of his vehicle to some extent even if he is not be held up by some other crossing stream.

The ITSM divides the road stretch into homogenous blocks with respect to road width, speed limit, curvature, gradient etc., A new set of block name "Intersection block" is included in this model. The intersection block is defined in terms of the intersection length and other geometric characteristics like number of lanes etc., An "Influence block" is introduced between the intersection block and adjoining homogenous road blocks. The influence block acts as a transition zone for the vehicles desiring to enter intersection from the road block, thus vehicles may reduce their speed and can also wait in queue.

the influence block. The change in the road width is accounted for also in the influence block. An influence block beyond the intersection (influence block on exit) is also introduced that accounts for the acceleration of the vehicle from the local speed to the free speed of the next road block. The intersection has three components :-

- (i) Influence block on entry
- (ii) Intersection block
- (iii) Influence block on exit

The length of the intersection block is based on the geometry of the intersection. While the length of the influence block at entry and exit are decided considering the geometry of road and vehicle characteristics. The length of the influence block on entry is more than that of the length of the influence block on exit; The block layout is as shown in fig.3.1

To differentiate between an intersection block (including its influence blocks) from the other homogenous road blocks, codes are assigned for each block in the road data file as below

```
code = 0   Road block
code = 1   Intersection block
```

Based on these code the relevant logic for the road block and

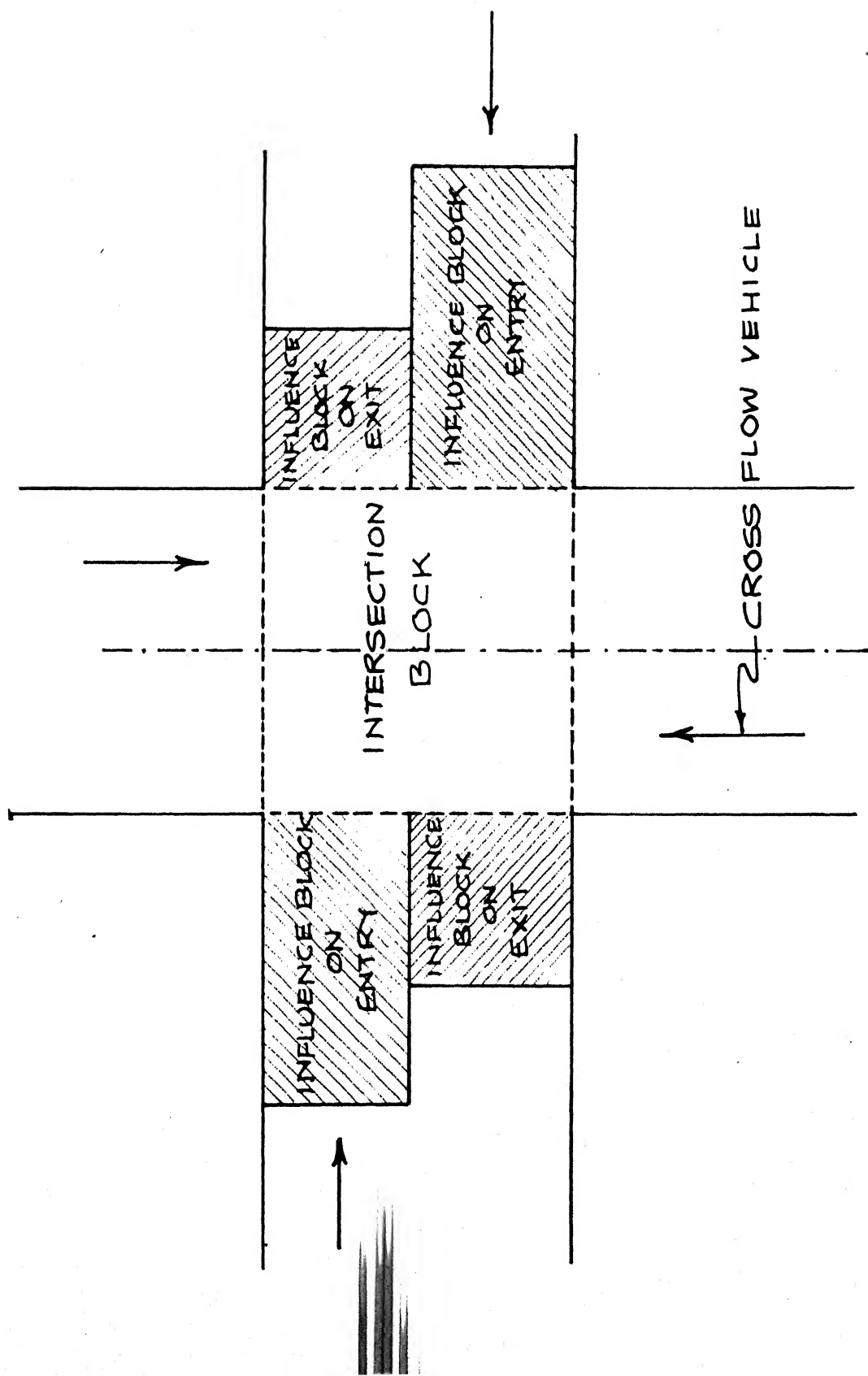


FIG. 3.1 - INTERSECTION CLASSIFICATION IN THE MODEL

intersection block are called.

3.4 Flow logic:

The flow logic for the vehicles entering an intersection consists of the manner in which each vehicle is going to react to the situation it encounters at the intersection, with respect to vehicles waiting ahead of it, the length of the intersection, the flow levels in the cross stream, intersection geometry etc., actually before entering the intersection.

The driver reaching the start of an influence block on entry has to make certain decisions before he ventures up any actions. These include

- Whether the vehicle has reached its destination. If it has reached destination whether to take a left or right turn. Left turning does not involve any interaction with cross flow.
- For straight and right turning movements crossing flow logic is adapted.

The crossing gap logic for a vehicle arriving at an intersection is described with reference to the fig.3.2.

It illustrates two cases

- (a) The movement of a vehicle through an intersection without any delay, denoted by vehicle 'i'.

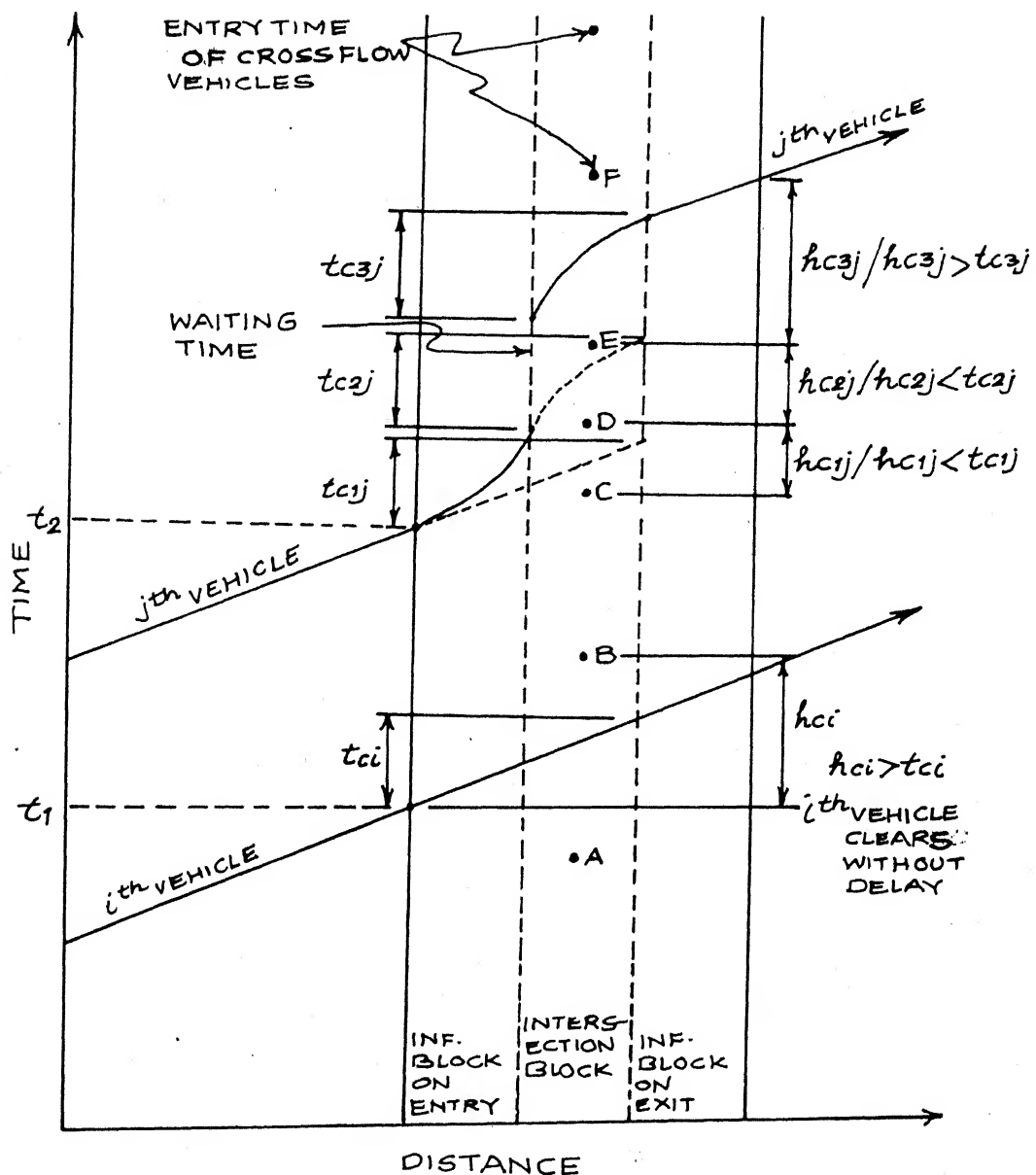


FIG.3-2 - FLOW LOGIC THROUGH INTERSECTION

- (b) Delay caused to the vehicle due to insufficient time headways between cross flow vehicles. This is detailed by vehicle 'j'.

The schematic logic for the case (a) above is as follows:

(i) Let the vehicle 'i', reach the start of an influence block on entry at time ' t_1 '. The vehicle then computes the time for clearing the intersection under local speed condition (t_{ci}).

(ii) The i th vehicle then checks for the time gap between the arrivals of cross flow vehicles. The time gaps for the cross flows are generated negative exponentially through a system procedure. It can be noted from the figure that the vehicle 'A' in cross flow has already crossed the intersection long before the i th vehicle reached the intersection. Now the i th vehicle checks for its ability to clear intersection before it is interrupted by the vehicle 'B'.

(iii) The test is carried firstly by calculating the time gap at the instant i th vehicle is at the beginning of the intersection and the position of the vehicle 'B'. Since this time gap (h_{ci}) is greater than the time for clearing intersection by the i th vehicle (t_{ci}), the vehicle clears the intersection without delay.

In case, for a vehicle which is subjected to delay at intersection, the flow behaviour is described as follows:

(i) Let the vehicle in description j th vehicle be at the beginning of an influence block on entry at time ' t_2 ', the time of clearing intersection by this vehicle with its local speed be ' t_{c1j} '.

(ii) The vehicle ' j ' then checks for the gap (h_{c1j}) provided by cross flow vehicles C and D, which is less than ' t_{c1j} '. So, the vehicle slowly decelerates its speed and reaches the intersection block and looks out for next headway.

(iii) The subsequent headway (h_{c2j}) between the oncoming vehicle E and the vehicle ahead D is also not sufficient for the j th vehicle to clear in time ' t_{c2j} '. Considering the j th vehicle waits for the next oncoming vehicle to provide the appropriate gap.

(iv) The j th vehicle then checks for the new time headway (h_{c3j}) between the vehicle F and its present position. If the time of clearance for the j th vehicle (t_{c2j}) is sufficiently less than (h_{c3j}) the vehicle accelerates from its waiting position to come out of intersection.

Until now we have considered the movement of a single vehicle clearing an intersection with or without delay. If suppose, there is a vehicle coming on to

intersection finds that there are already few vehicles waiting to find gap, this particular vehicle then decelerates its speed and joins the waiting queue in the influence block on entry. As soon as sufficient time gap in the arrivals of cross flow vehicles is found, the vehicles waiting in the queue are processed on the First In and First Out basis.

The cross flow logic discussed above holds good when there is no interaction between the adjoining vehicles in the stream. But if there is any interaction it is checked before the cross flow logic is activated.

The conditions that are necessary for the vehicles in order not to interact with the vehicles moving ahead in the intersection are as follows:-

- The headway between the current vehicle and the vehicle ahead should be greater than the head length of the catching up vehicle plus tail length of the vehicle that is caught up.

(Head length indicates the distance required to slow down with given retardation to the speed of preceding vehicle.

Tail length is chosen so that the vehicles in a queue are separated by given time intervals)

- Speed of the vehicle ahead is more than the current vehicle.

The intersection of flow logic impose control over the vehicles by not permitting any lane changes and overtaking operations during their movement through intersections. In processing these logic's and also the queue disciplines, CLASS features of SIMULATION and SIMSET available in SIMULA are used.

3.5 Data structure:

The overall structure of vehicle as it flows on the road system is represented as in fig.3.3.

VEH - When the entry time of an vehicle equals the clock time, it is activated.

WAITSUFTIME - Waits for certain time with respect to other vehicles in the traffic stream before joining track(2).

VEHPROCESS - The vehicle starts and proceeds to its destination.

INTERSECTION

AND - Before actually the vehicle starts it checks NON-INTERSECTION whether the road block is an intersection block or not. If it is an intersection block or not. If it is an Intersection block the process of vehicle takes place through procedure INTERSECTION or else the vehicle

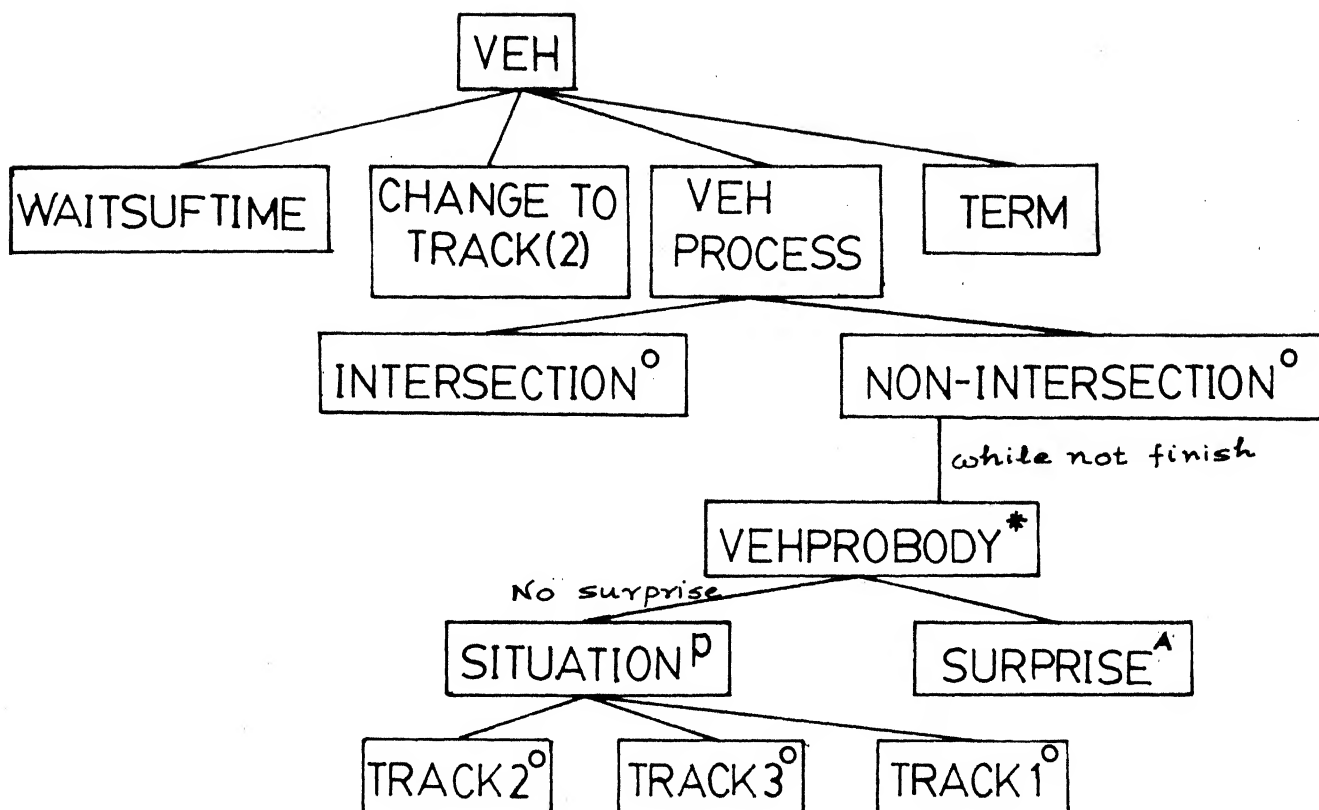


FIG. 3.3

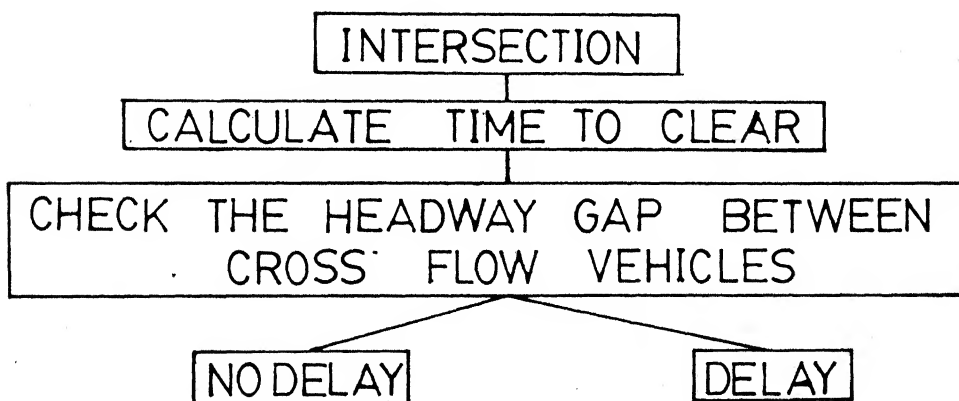


FIG. 3.4

treats the block as an ordinary road block.

VEHPROBODY

- If the vehicle is on a road block instead on intersection, this process is activated in which all the general procedures required for an vehicle to move on a road are described. It consists of a choice between
- SITUATION - It consists of three alternatives

TRACK 2

TRACK 3

TRACK 1

- SUPRISE - It is assumed before this process that the vehicle is not subjected to any surprise by any other vehicle. If this assumption proves wrong the vehicle is made to undergo this process.

TERM

- This process terminates the vehicle on reaching its destination.

The program structure for an vehicle when it enters an intersection is as shown in fig.3.4.

When a vehicle entering an intersection finds

that its movement is not going to be hindered by any cross flow vehicle, it clears the intersection without delay. The program structure for this type of vehicle is represented as in fig.3.5.

For a vehicle which is delayed due to insufficient gap for clearing the intersection the structure is represented as in fig.3.6.

The description of the labels which are represented in the blocks is as follows:-

- INTERSECTION - When the vehicle enters an intersection block, this procedure is activated.
- CHECK - It is a selection procedure, it checks for larger value of cross flow headway to clear vehicle with or without delay.
- NODELAY - The vehicle which is not constrained by cross flow vehicles clears the intersection through this procedure.
- DELAY - When a vehicle is delayed due to insufficient gap provided by cross flow vehicles this procedure is activated.
- CROFLHDGTITER - If the time headway between vehicles in cross direction is greater than time to clear an intersection, this is executed.

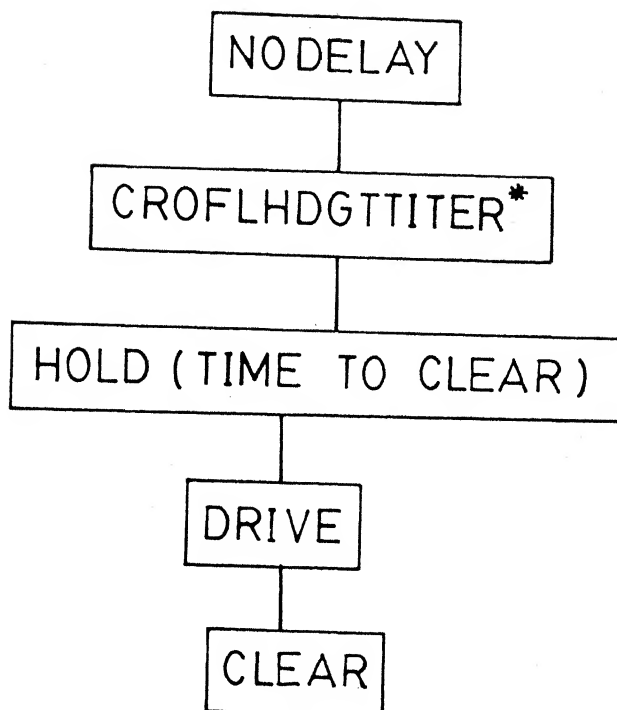


FIG. 3.5

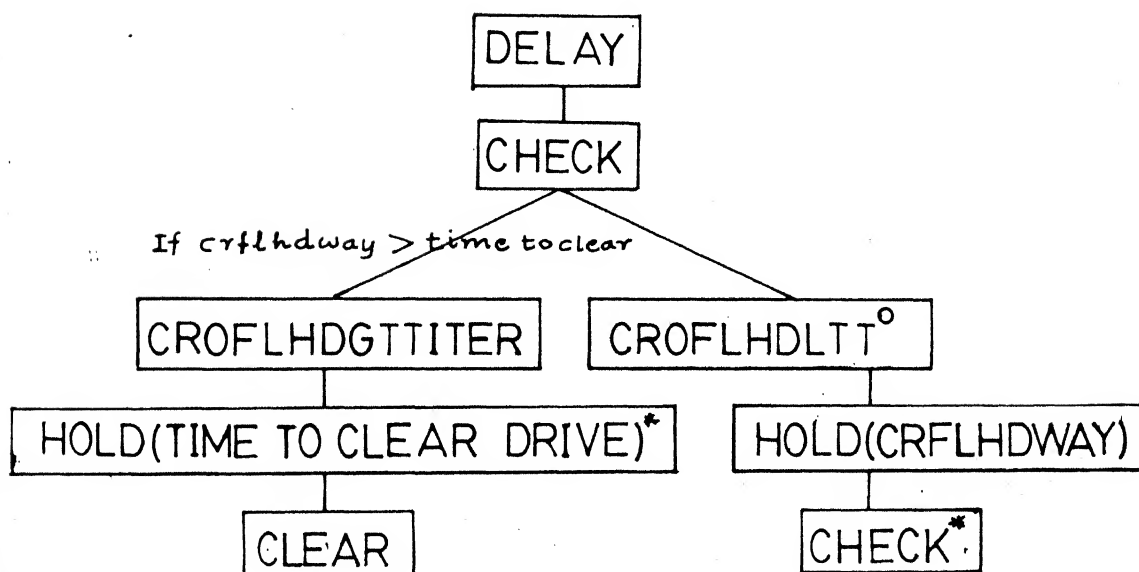


FIG. 3.6

- HOLD - Updates the time for the vehicle to clear an intersection.
- DRIVE - The model data is updated and vehicle drives along the road.
- CRFLHDLTT - If there is no sufficient headway between cross flow vehicles this procedure is executed.
- HOLD(CRFLHDWAY) - It holds the time headway and awaits for next headway.
- CHECK - checks for headway is greater than or equal to time for clearing an intersection by an vehicle.

3.6 Model inputs:

The inputs to the model consists of two types deterministic and stochastic. The deterministic inputs include road geometry, vehicle performance characteristics etc. and the later includes driver characteristics e.g. desired speed, overtaking gap acceptance etc., and traffic characteristics e.g. headway distributions.

3.6.1 Road data:

The model considers the road to be simulated as consisting of a series of consecutive road blocks and a sight distance function in each direction of travel. Each

road block is homogenous with regard to road geometry (road width, surface type, auxillary lane, longitudinal slope, horizontal curvature) and traffic regulations (speed limit, overtaking restrictions, intersection code). The inputs to the model for each block and in each direction are as follows:

- Distance coordinate of start of block
- Length of the road block
- Median speed
- Radius of curvature
- Slope
- Overtaking restriction
- Lane code
- Intersection code
- Distance co-ordinates of the change of sight length
- Sight distance

3.6.2 Intersection data:

(a) Intersection length - The block lengths which are given as a input to the model from the road data file represents the length of an homogenous block. For the intersection the length includes (influence block on entry + intersection block + influence block on exit).

(b) Intersection code - This code is also given as a input

to the model from road data file which consists of road blocks having either of the two (0 or 1) code numbers assigned to them. The 0 code suggests it is a road block and code 1 represents it to be an intersection.

(c) Cross flow vehicles - The generation of time headways between crossing vehicles at the intersection is carried out by activating a system defined procedure. The mean rate of arrivals and a seed value is given as inputs to generate the time headways.

3.6.3 Vehicle data:

The inputs to the model with regard to driver vehicle attributes consists of

- Identity number
- Vehicle type
- Direction of travel
- Entry block co-ordinate
- Entry time
- Entry speed
- Exit block co-ordinate
- Basic desired speed
- Power-Weight ratio

These attributes can be obtained either from the field observations or by the generation of vehicle attributes from

the aggregated vehicle data. In the present study, the vehicle attributes are generated from the aggregated vehicle data are used as inputs. Inputs to the generation model are traffic flow and its composition for each direction. Identity number is trivially generated. Vehicle type is sampled from the traffic composition. The headway at entry is determined taking into account the traffic flow and overtaking opportunities. Entry speed is determined by the road geometry for the free flowing vehicles and the speed of the leading vehicle for the constrained vehicles. Basic desired speed of a vehicle is generated out of the basic desired speed distribution of that type, observed spot speed and the recorded journey speed. Power-Weight ratio is sampled from its distribution and a check is made to ensure that the vehicle can maintain its basic desired speed on a flat road with this p-value. These distributions have been calibrated in ITSM.

3.7 Model outputs:

The outputs from the model are recorded in two files **INTER** and **EVENT**. The **INTER** file consists of all the events related to processing of an vehicle through an intersection. The events that are recorded in an **INTER** file are:-

- Identification no. of vehicle
- Vehicle type to which the vehicle belongs
- Time of entering the influence block on entry
- Waiting time for the vehicle at the intersection due to delay, if any
- Time to clear intersection
- Exit time from influence block on exit.

It must be seen that in the further work that is to be carried out on this model, all the events relevant to an intersection are recorded in an EVENT file, which possesses a detailed description of the vehicle from the time of its entry into road stretch to the point of its exit. The outputs that are noted down in an INTER file are further processed to obtain total number of vehicles and their total waiting time for each vehicle type in order to calculate mean and standard deviation of waiting time for each vehicle type.

The output obtained in this form will be helpful to a traffic planner in case studying the traffic behaviour at an intersection to take up any engineering measures thereupon.

The event file contains a record of each event in the simulation run. These events are recorded in chronological order and can be used for further processing.

Simulation results recorded in the event file are processed further to obtain the following outputs :-

(a) Statistical distribution at spots:

- Spot speed
- Time headway
- Queue length
- Queue frequency

(b) Statistical distribution between spots:

- Journey speed
- Journey time
- Number of overtaking/passings

(c) Traffic description characteristics along the road:

- Percentages of vehicle km. as
 - * freely moving
 - * queuing
 - * overtaking/passing
- Traffic time delay
- Traffic fuel consumption

(d) Traffic safety concepts:

- Risk exposure
- Conflict situations

(e) Other traffic effects:

- Traffic noise

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- Traffic pollution

These outputs can be used for model validation to estimate the benefits accruing from improvements of a road geometry and establish generalized relationship between traffic flow and geometry.

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4.0 CHAPTER

MODEL RESULTS

4.1 Introduction:

The ITSM has been calibrated and validated with the field observations for plain, rolling and hilly terrains at different levels of service. As the simulation results show close agreement with the field observations, it can be said that the model possesses the potential for representing real life system of Indian highways. It is quite a complex model system.

In the present study a new submodel for traffic flow through intersections is incorporated and the detailed flow logic for the same is given in chapter 3.0. The main objective of the study was to incorporate this logic structure into the existing model; and test the program for this logic, by simulating the traffic flow on a road section of 7 km. length with 2-3 intersections of different characteristics. The detailed outputs of all the events related to the intersection flow logic was thoroughly checked. The program finally emerged satisfactorily with this logic.

However, the detailed analysis of simulation results for the entire road system is not taken up due to

constraints and is out of scope of this work. But the results that are recorded with respect to an intersection were analysed and are found to be satisfactory. It is suggested that the actual field observations should be carried out with respect to the headways in cross stream and actual speed profile of vehicles moving through intersections. These observations will help to calibrate various parameters and facilitate the validation against the field observations. The logical structure developed presently is such that any changes to the model in future can easily be incorporated.

4.2 Design of simulation experiments:

Initial testing of the model for the traffic flow through intersections was done by recording the detailed output of each individual vehicle as they pass through an intersection. The output with respect to each vehicle consists of all the relevant events like entry time, waiting time, time of exit and also the time of entry of other interacting vehicles are recorded and studied for further analysis. Once it was confirmed that the program is as per the flow logic, it was decided to conduct some simulation experiments on the model thereupon under different input levels. Due to various constraints of computation power available, a few simulation runs were made.

The flow process through an intersection depends upon :-

- Intersection geometry
- Flow pattern on the road i.e. the type, position and speed of all the vehicles moving on the road under consideration.
- Flow pattern occurring on the crossing highways.

To have a fair idea of the traffic behaviour at the intersections, the simulation experiments were performed to cover a wide range of the input parameters.

A 7.0 km. long stretch of road with an intersection of 50.0 mt. length between road co-ordinates 3.45 km. to 3.50 km. in direction 1 and 3.5 km. to 3.55 km. in direction 2 was considered for simulation. A traffic composition consisting of cars (25%), HMV (60%) and two wheelers (15%) was taken for experimental design. The traffic flows on the road were varied from 100 veh/hr. to 400 veh/hr. at an interval of 100 veh/hr. For each of these four levels considered the cross flow levels were varied from 360 veh/hr. to 1200 veh/hr. with mean headways of 10,8,5,3 sec. respectively. A set of 16 simulation runs were conducted on the model to yield the results considering the above input levels.

4.3 Analysis of simulation results:

The performance measure with regard to the vehicles in terms of waiting time at intersection was considered for analysis. The mean and standard deviation of these waiting times for each vehicle type was analysed and the results are given in table 4.1.

The frequency distribution of the waiting times for different levels of traffic flow are given in fig.1, 2, 3. Fig.1(b) shows that when the mean cross headway is 10 sec. (cross flow level of 360 veh/hr.) at a traffic flow level of 200 veh/hr. the waiting time for most of the vehicles is less than 20 sec. However, when the traffic flow is considered to 400 veh/hr. for the same cross flow level the waiting times of the vehicles has considerably increased. This increase in waiting times is due to the interaction between the vehicles on the road under simulation. This interaction takes place when the vehicles have a tendency to form a platoon at the intersection. A similar trend is observed in fig.2(a),2(b) and 3(a),3(b) where the cross flow levels are 450 veh/hr. and 720 veh/hr. respectively. The only difference is that the mean cross headway has reduced (8 sec.) the vehicles get less opportunity to clear the intersection. When the cross flow still further increased to

Traffic flow (veh/hr)	Cross flow (veh/hr)	Mean waiting time (sec.)		
		cars	HMV	cars and HMV
400	360	35.29	9.89	17.32
	450	110.08	17.02	44.25
	720	186.07	33.45	78.11
	1200	194.97	214.56	208.82
200	360	4.76	8.05	7.08
	450	11.80	16.83	15.35
	720	34.74	44.46	41.61
	1200	170.20	136.29	146.15
100	720	22.55	40.60	35.32
200		34.74	44.46	41.61
300		37.05	49.89	46.13
400		186.05	33.45	78.11
100	450	12.46	13.06	12.88
200		11.80	16.83	15.35
300		9.98	19.10	16.43
400		110.08	17.02	44.25

Table 4.1

720 veh/hr. the vehicles on the simulated road may have to wait longer to find an acceptable gap in the crossing stream. The results in fig.4(a),4(b) shows that for a cross flow level of 1200 veh/hr. the waiting times for the vehicles are very high. The mean waiting times for the vehicles at different flow levels are given in fig.5(a),5(b). The values indicate that for a particular traffic flow level the waiting time increases with the increase in cross flow level. Beyond a certain cross flow level the increase is very sharp. Also the mean values of waiting times has decreased from 214 sec. to 136 sec. With the decrease in flow level from 400 veh/hr. to 200 veh/hr. respectively.

The plot in fig.6 shows that up to a traffic flow of 300 veh/hr. the traffic is almost under free flowing condition and there is not much increase in the waiting time. However, with the increase in traffic flow to 400 veh/hr. waiting times have sharply increased. This indicates that interaction has started building up between the vehicles moving on the road under simulation.

As it is a common observation that, with increase in flow levels either on crossing road or road under simulation, the delay for the vehicle increases. The simulation results obtained also prove

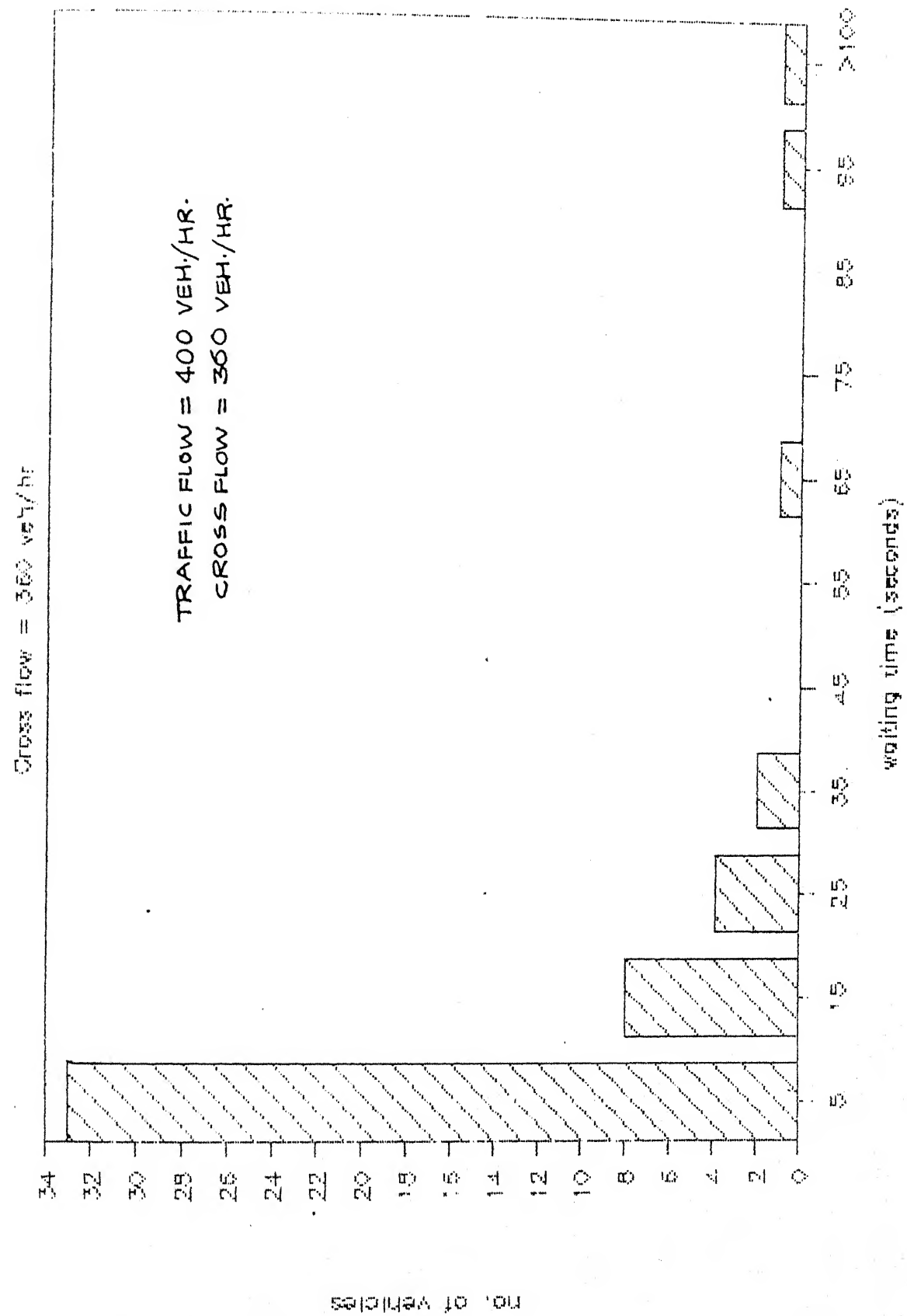
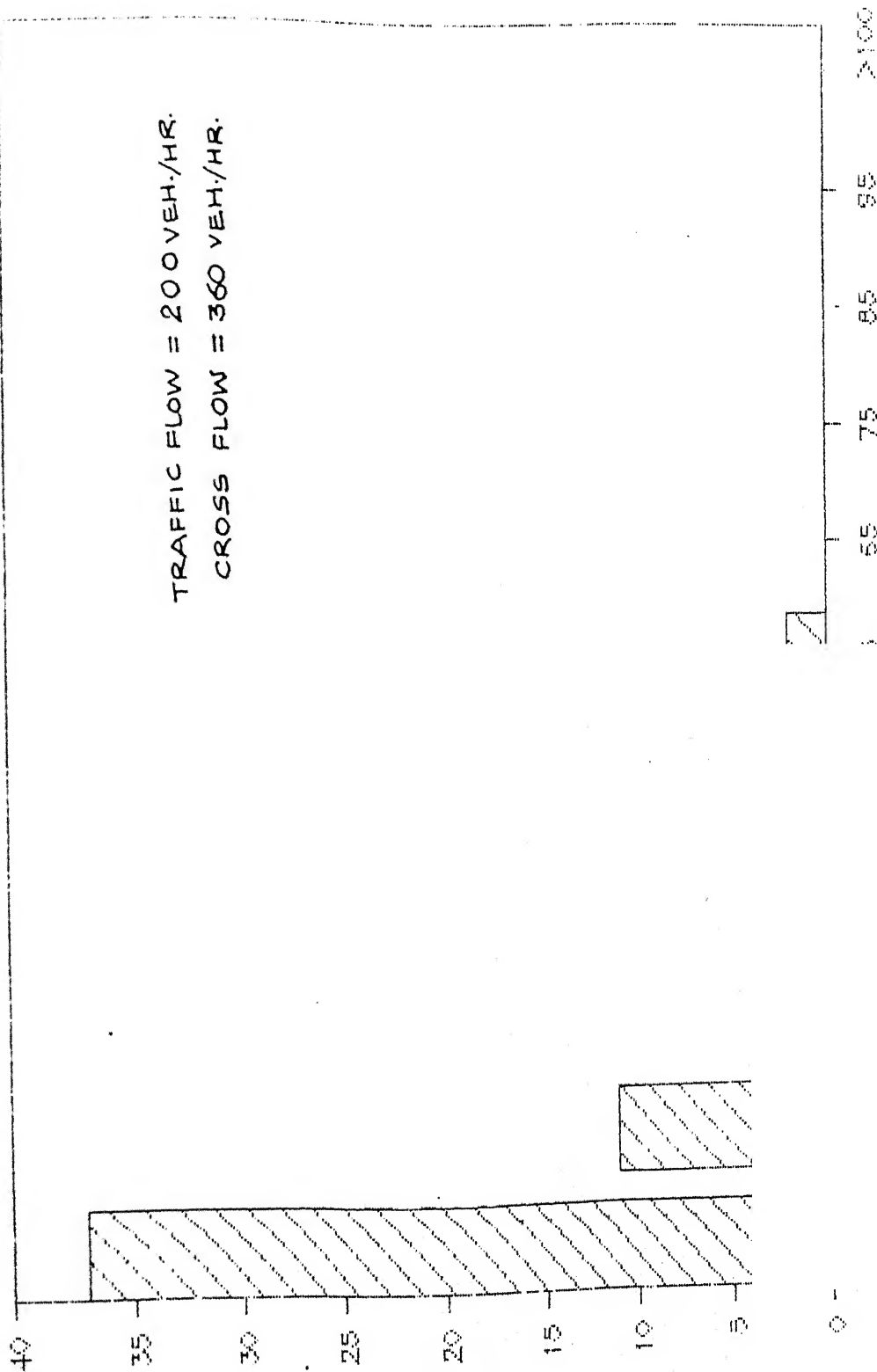


FIG. 1(a) - FREQUENCY DISTRIBUTION OF WAITING TIMES

Gross flow = 350 veh/hr

TRAFFIC FLOW = 200 VEH./HR.

CROSS FLOW = 360 VEH./HR.



seconds

DISTRIBUTION OF WAITING TIMES

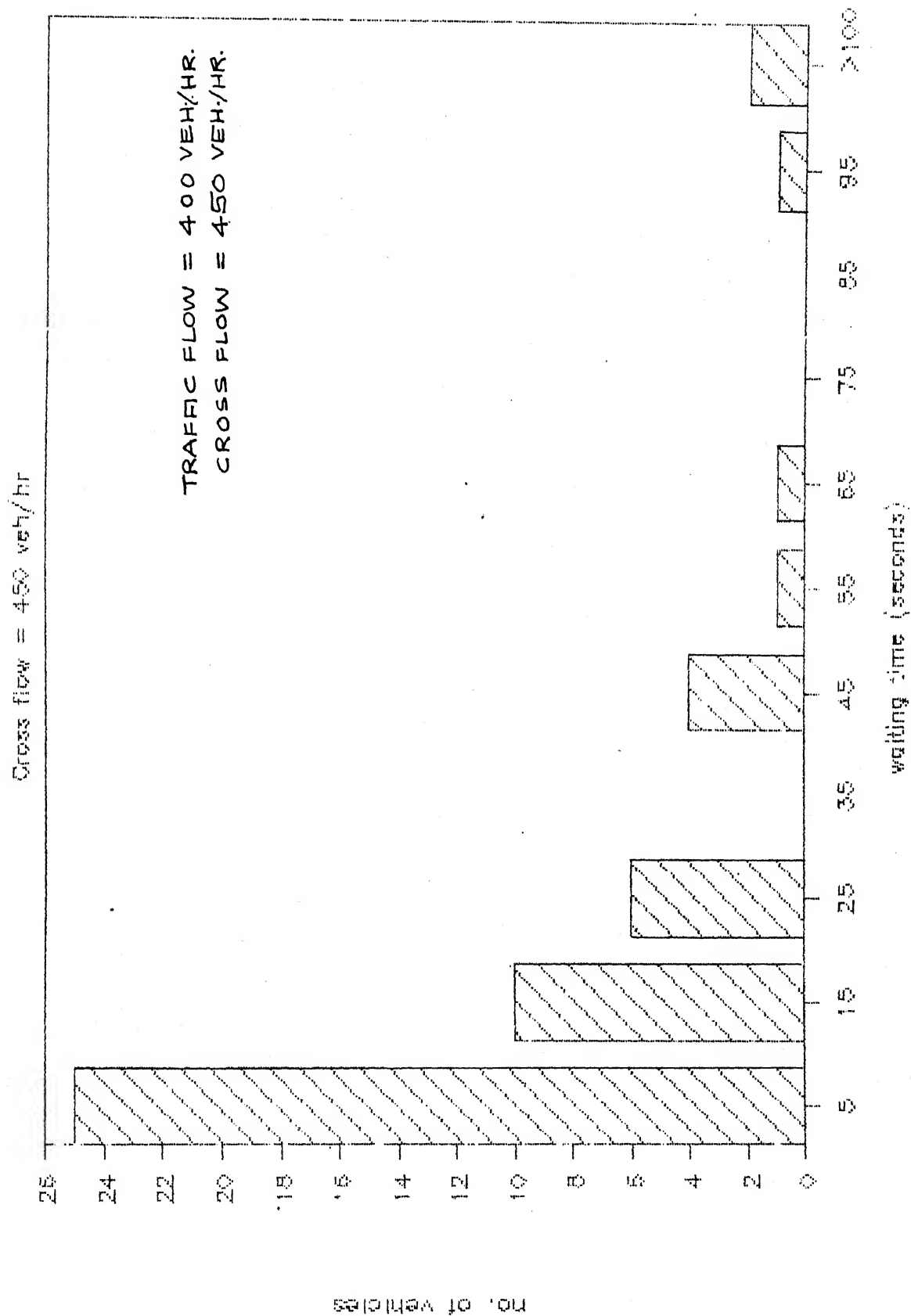


FIG. 2(a) - FREQUENCY DISTRIBUTION OF WAITING TIMES

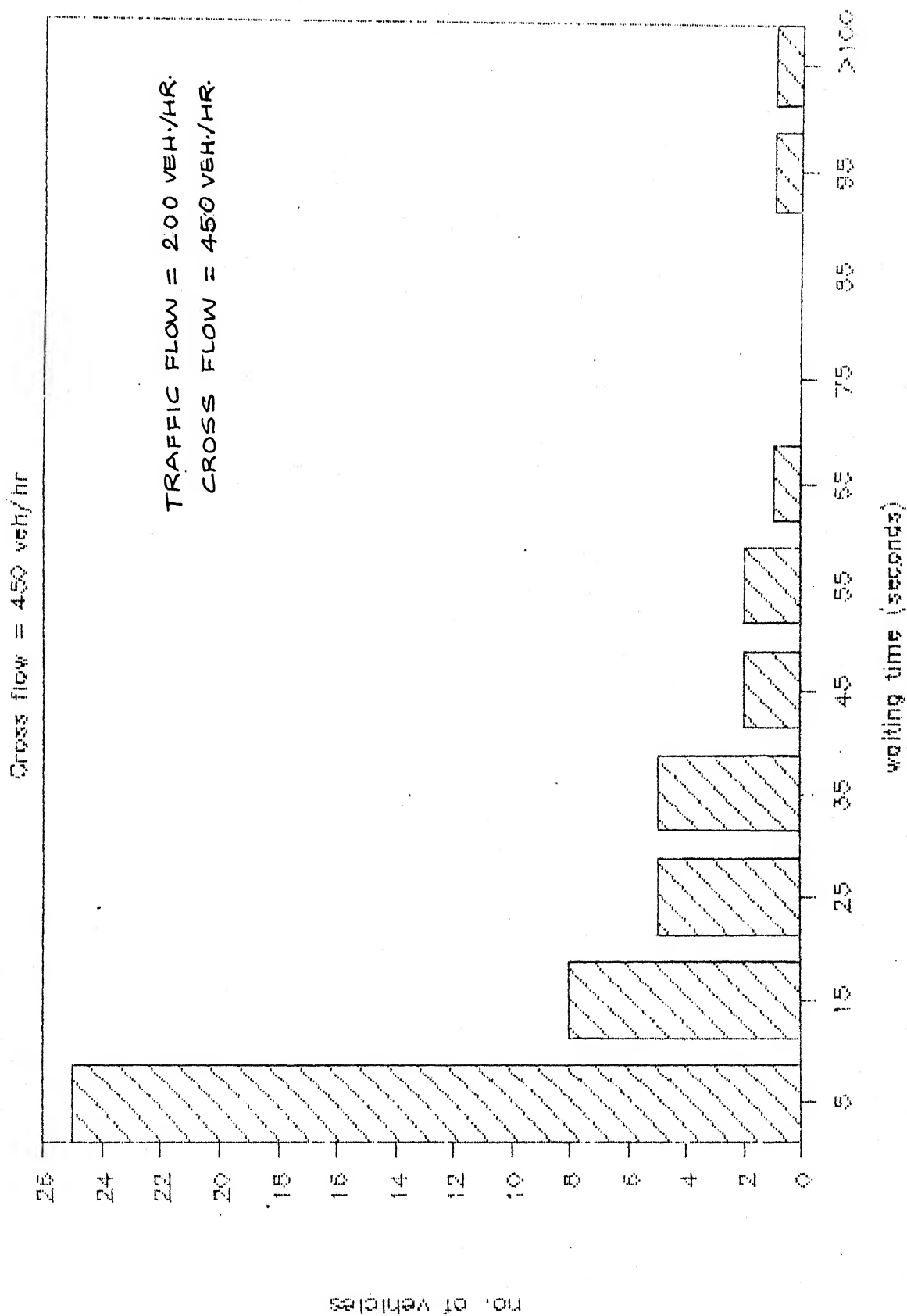


FIG. 2(b)- FREQUENCY DISTRIBUTION OF WAITING TIMES

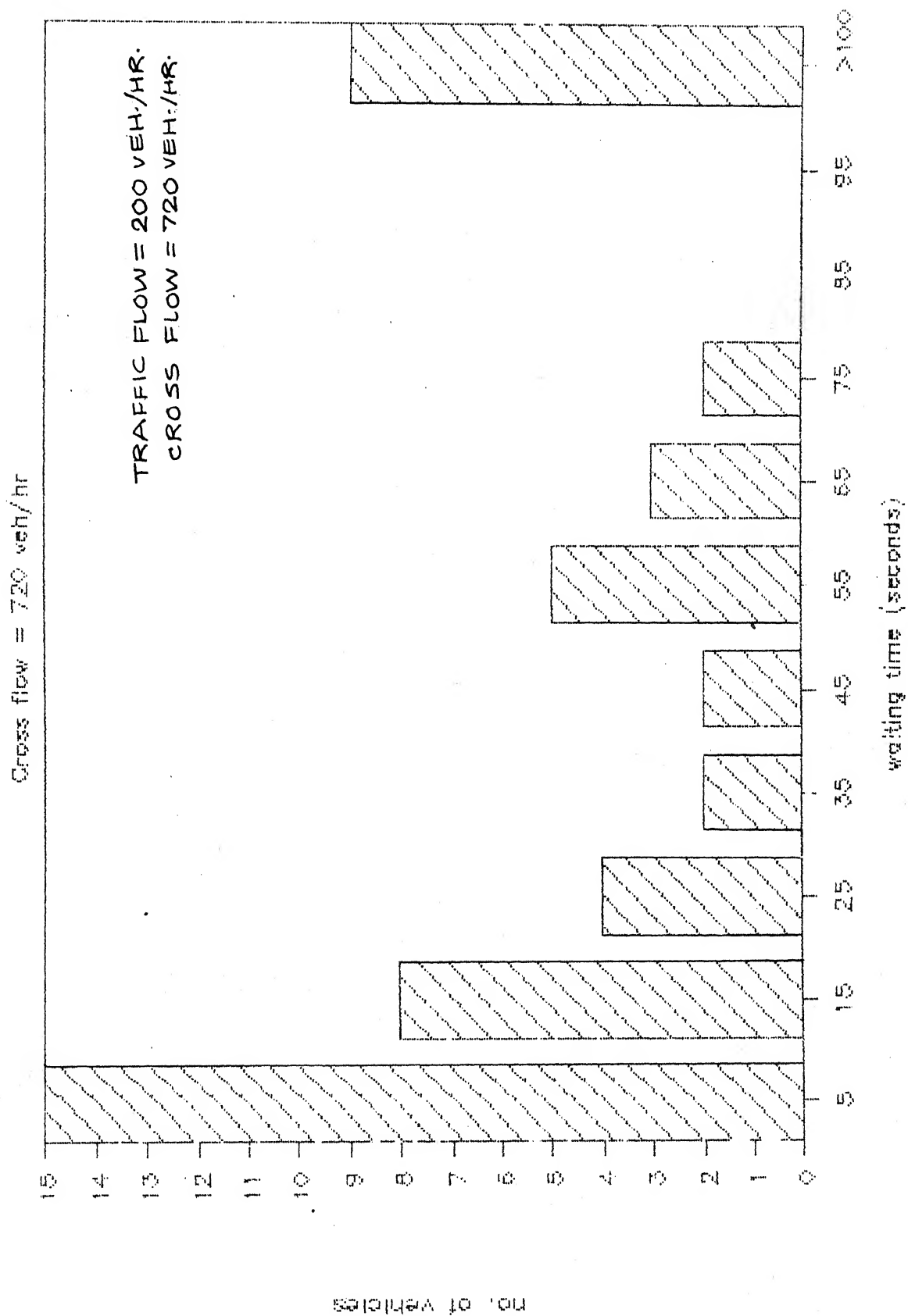


FIG. 3(b) - FREQUENCY DISTRIBUTION OF WAITING TIMES

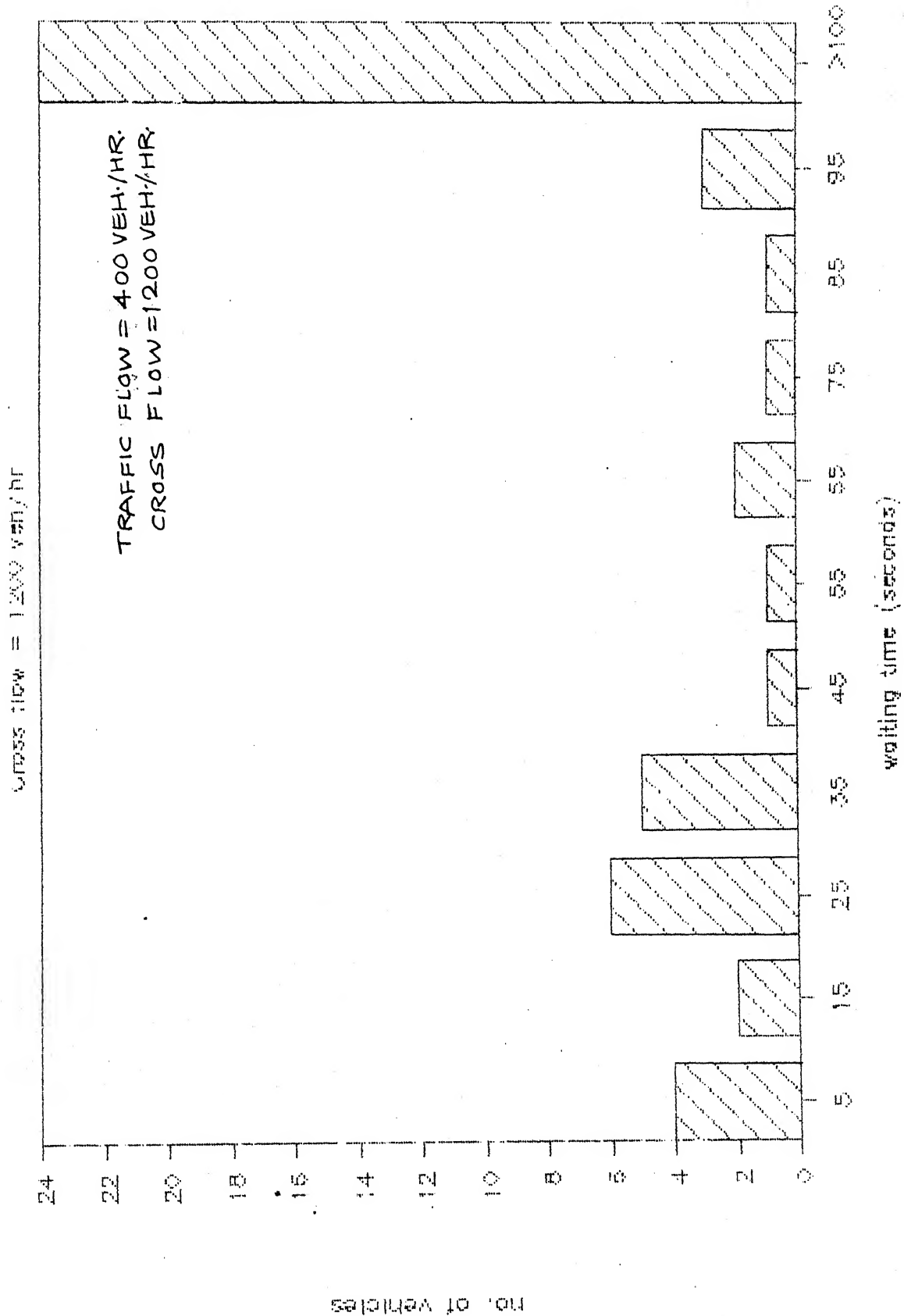


FIG. 4(a) - FREQUENCY DISTRIBUTION OF WAITING TIMES

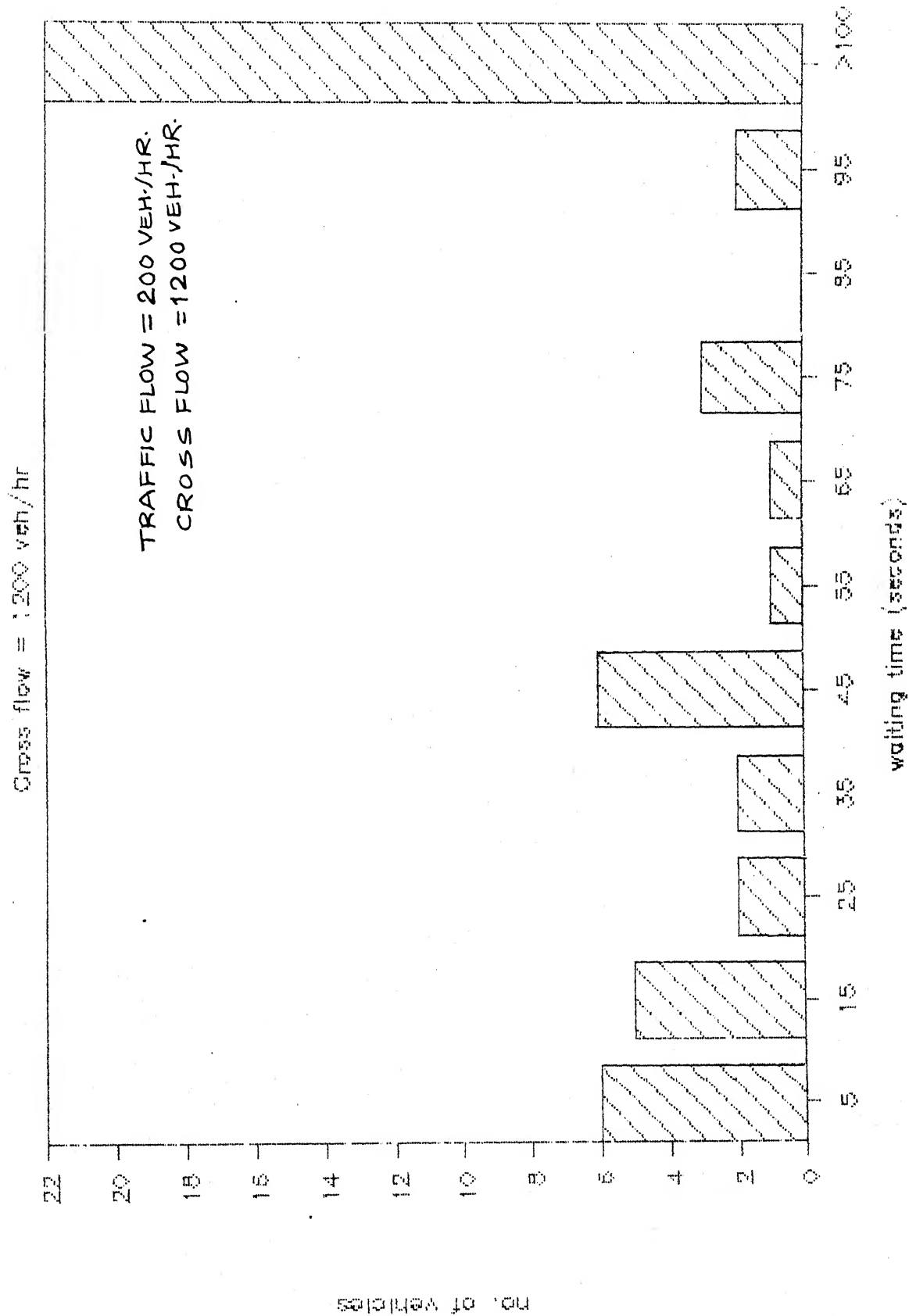


FIG. 4(b)- FREQUENCY DISTRIBUTION OF WAITING TIMES

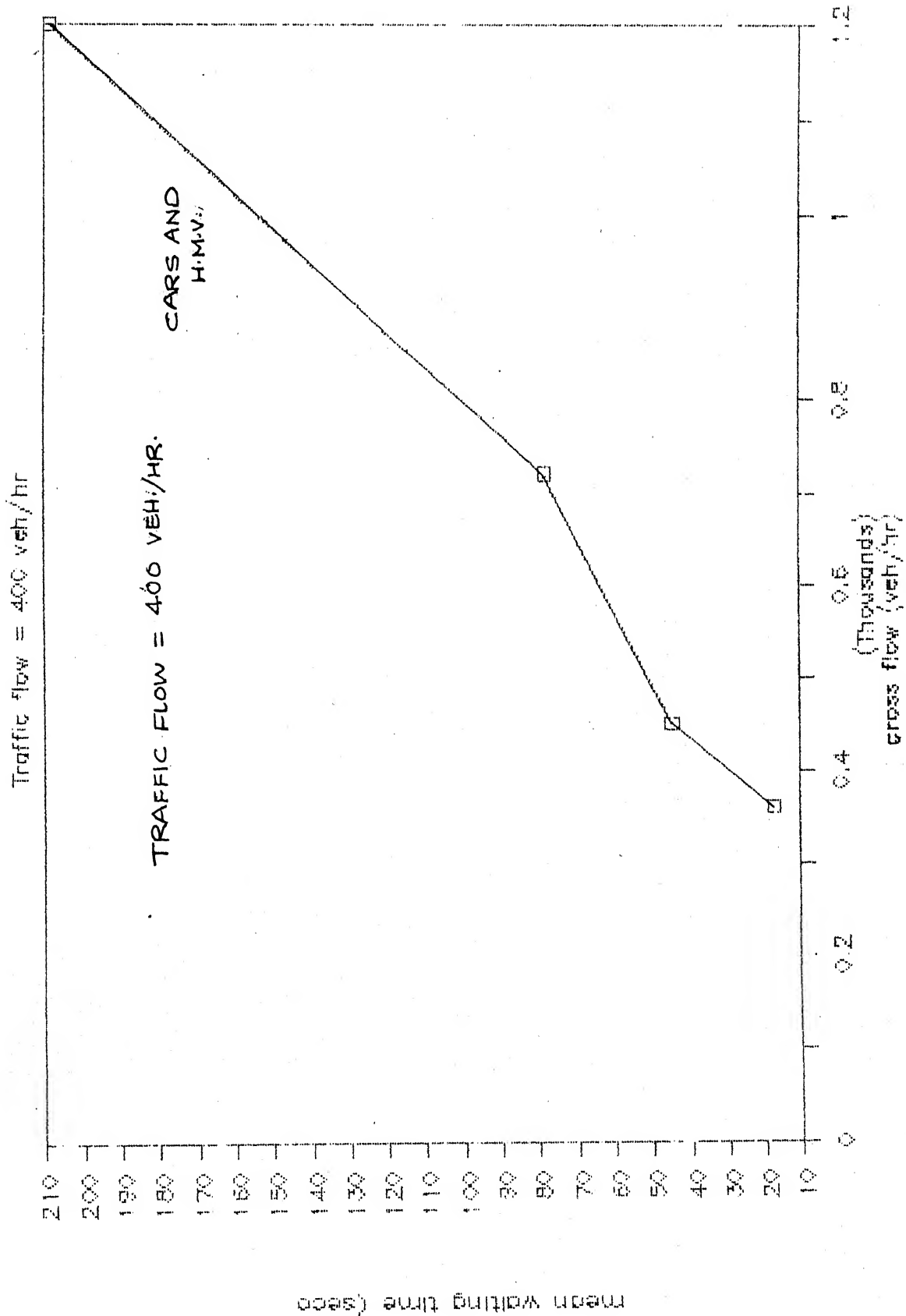


FIG. 5(a) - MEAN WAITING TIME FOR DIFFERENT CROSS FLOW LEVELS

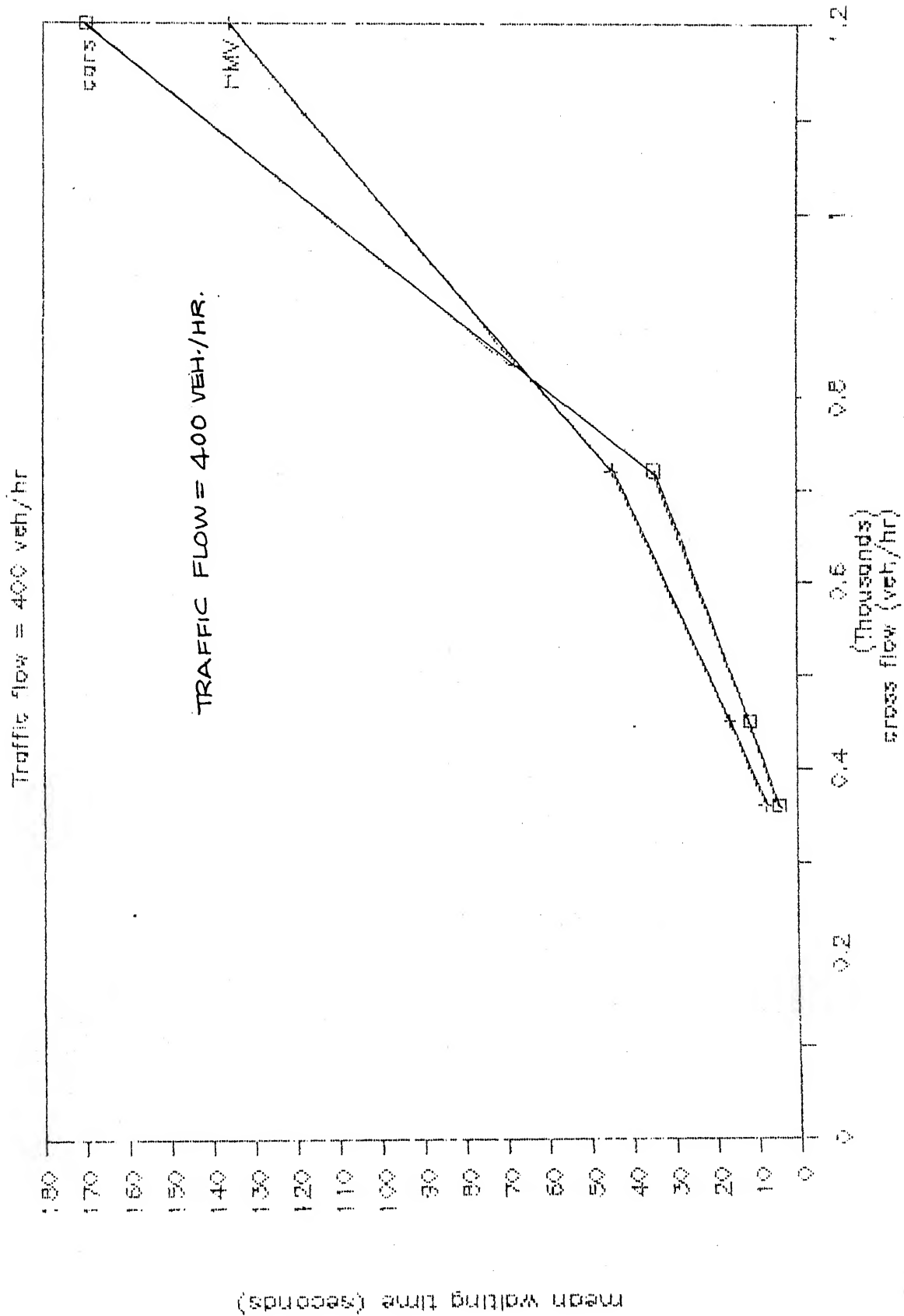
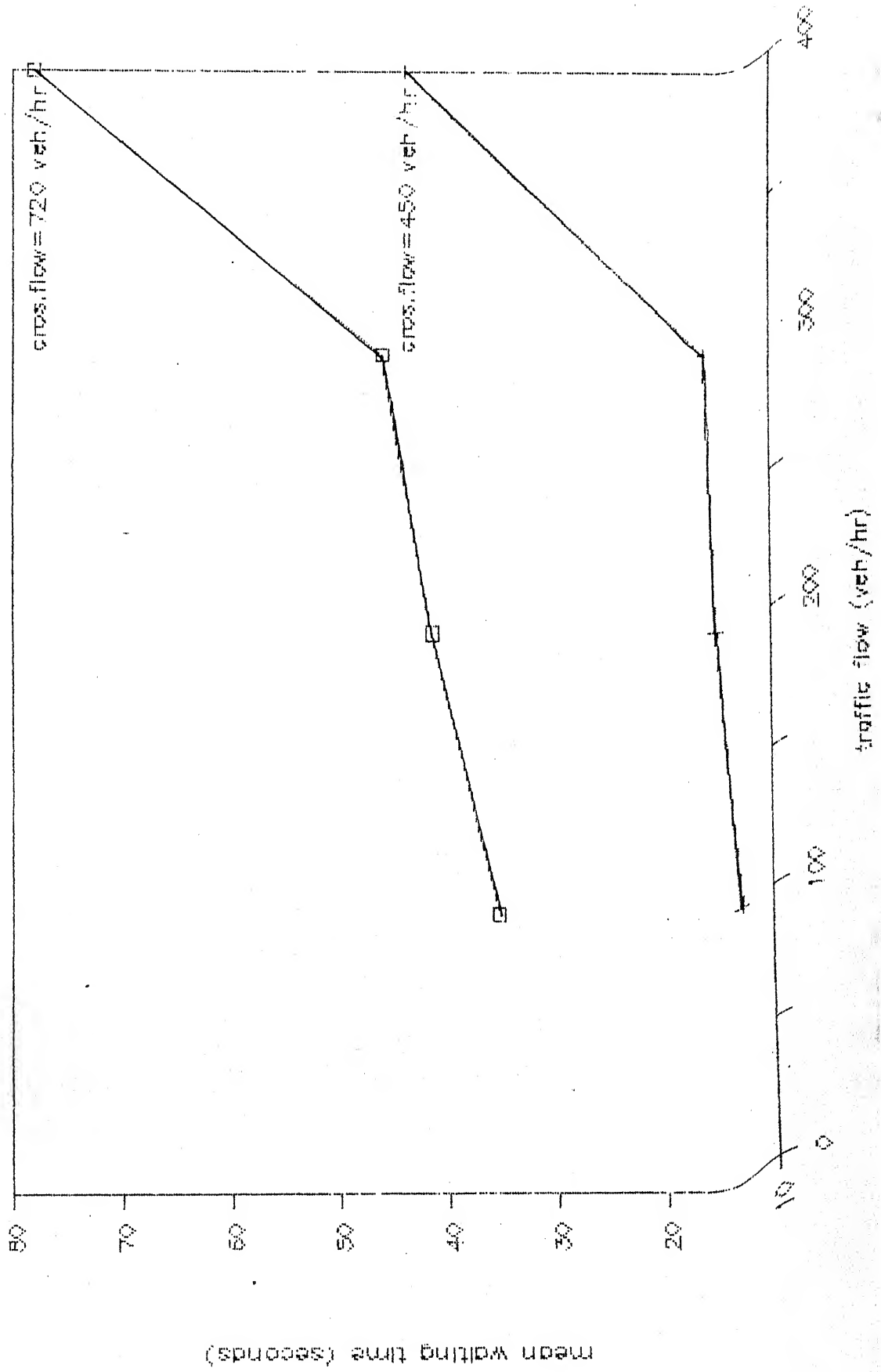


FIG. 5(b)- MEAN WAITING TIME FOR DIFFERENT CROSS FLOW LEVELS



point, giving the impact of uncontrolled intersection with regard to delay caused at intersection. The results also provide useful information in understanding the traffic flow system at intersections.

4.4 Utility of the model:

The simulation results can be used to estimate the capacity of unsignalized intersections of various levels of service (in terms of waiting times).

The impact of alternative traffic management strategies of signalization, provision of grade separated intersections can also be evaluated through the simulation of the system. It provides the planner an insight into the problem and to determine his action in taking long term or short term traffic engineering measures. The model also helps in identifying the zone of influence for the intersection, providing a comprehensive picture of the things that are occurring outside the sphere of the actual intersection.

4.5 Conclusions:

The model for traffic flow through intersections developed in the present study simulates the flow of cars, HMV, and two wheelers as they pass through an intersection. The program structure namely JSP which is used in making up of the original ITSM is also implemented in the

new submodel so as to be incorporated into the ITS model without distorting its initial structure. The programming work is carried out in SIMULA67.

Some of the features of the model developed in this work include:-

- takes note of each individual vehicle that comes to pass through an intersection.
- distinguishes an intersection from a non-intersection.
- captures the events that takes place within an intersection environment.

4.6 Future work:

The further work that can be carried out for model development should include :-

- calibration of model parameters through field data.
- Validation of the model under different input levels.

These can be done through comprehensive field data.

Once all these conditions are met the model can be applied with greater confidence on any section of Indian road system.

REFERENCES

1. Carlsson, A, Gynnerstedt, G, Westerlund, B, A model for the Monte Carlo simulation of traffic along two-lane single carriageway rural roads Meddelande 43.
National Swedish Road and Traffic Research Institute 1977.
2. Birtwistle, Dahl, Myrhaug, Nygaard.
Simula Begin
Auerbach Publishers Inc. Philadelphia Pa 1973.
3. Ingevaldsson, L,
JSP A practical method of program Design.
4. Brodin, A, Gynnerstedt, G, Levander, G,
A program for the Monte Carlo simulation of Vehicle traffic along two-lane rural roads.
Meddelande 143.
National Swedish Road and Traffic Research Institute 1980.
5. National Swedish Road and Traffic Research Institute,
Report No. 322A, Sweden, 1983.
6. Marwah, B.R.,
A simulation model of two-lane and four-lane highways for Indian traffic Conditions, Central Road Research Institute, New Delhi, 1983.
7. Mitrani, I,
Simulation techniques for discrete extent systems,
Cambridge University Press, Cambridge, 1982.